~~Absorbing~~ Employing a Markov Chain Model~~s~~ to Determine Optimum Process Target Levels in Production Systems with Dual Correlated Quality Characteristics

Comment [A1]: Please note the changes to the title. If acceptable, please use the revised title when referring to this paper from now on.

The title page should also include:
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Abstract

Cutting costs and improving overall efficiency is essential f~~F~~or any manufacturing organization to compete ~~effectively~~ efficiently o~~i~~n the global marketplace~~, cutting costs and improving overall efficiency is essential~~. In this context~~A~~, a single-stage production system with two independent quality characteristics and the ~~different~~ costs associated ~~with~~ with each quality characteristic that falls below a lower specification limit (scrap) or above an upper specification limit (rework) are~~is~~ ~~presented~~ considered in this ~~paper~~study. The ~~amount~~ numbers of reworks and scraps ~~are~~ are assumed to be dependent~~ing~~ on ~~the~~ process parameters such as process mean and standard deviation. Therefore,~~thus~~ the expected total profit is significantly dependent on ~~the~~ process parameters. To determine process means, t~~T~~his ~~paper~~ study develops a Markovian decision-making model ~~for determining the process means~~. Sensitivity analysis~~zes~~ is then performed to validate the results~~,~~ and a numerical example ~~is~~ given to illustrate the proposed model. The results show~~ed~~ that ~~the~~ optimal process means ~~extremely effects~~ affect ~~on~~ the quality characteristics' parameters significantly.

Comment [A2]: According to the author guidelines of the *Journal of Economics*, the abstract should be between 150 and 200 words. Right now, it's well below the minimum number. The Abstract should discuss the implications of the stud—specifically who will benefit from the findings?

Keywords: Markov Chain, Process Mean, Bi-V~~v~~ariate Normal Distribution.

JEL Classification:

Comment [A3]: An appropriate number of JEL codes should be provided.

1. Introduction

~~In this paper, a production process with two quality characteristics is considered. A Markovian model is developed where defective items consisting of scrap and rework are produced, detected, and discarded during the process of manufacturing. To optimum the expected profit, scraps and reworks costs are considered in the model which is discussed in section 2. The optimum process means for two quality characteristics is determined in section 3. A sensitivity analysis is performed by varying the cost parameters, such as scrap cost, rework cost in section 4. A numerical example is provided in section 5.~~

Comment [A4]: I have moved the first paragraph to the end of the introduction, as it addresses the scope of the paper and its structure.

In the manufacturing process, a~~A~~ product ~~in the manufacturing process usually~~ should generally satisfy a set of specifications. One of the important parameters ~~of a~~in terms of quality characteristics is the product target mean. As such, t~~T~~he problem of selecting ~~the~~ optimal target means has~~-~~ attracted the attention of researchers for several years,~~been an impotent research area for many years~~ as~~.~~ d~~D~~etermining the optimal target mean of a quality characteristic is financially important. ~~Basically~~In other words, if either positive or negative ~~deviations~~ variations ~~in the two directions~~ of quality characteristics in relation to a threshold have equal costs, then the optimal process mean ~~of the process~~ is represented by the median~~iddle point~~ of the tolerance limit~~s~~. ~~However~~As expected, when the ~~deviation~~ variation of a quality characteristic in one direction is ~~more~~ costlier~~y~~ than ~~in~~ the one in the opposite direction, the optimal process mean ~~of the process~~ is not represented by the median ~~the middle point~~ of the tolerance limit~~s~~ (Abbasi et al. 2006).

Comment [A5]: I have used a more context-specific term here.

~~In a manufacturing process~~Typically, d~~, the d~~efective items are usually produced ~~with ~~along with good-quality finished goods during the manufacturing process. As such, a r~~R~~ework process is necessary to convert those defective items~~s~~ into finished ~~goods~~products. As ~~many ~~most systems are not perfect, some scrapped items are~~is~~ expected to be produced as a result of the~~during this process of~~ manufacturing and rework processes. As t~~T~~hese scraps and reworks can reduce profitability,~~.~~ ~~Many ~~numerous statistical tools have been developed to maximize the potential profit for an item in the production setting~~s~~.

For instance, Rahim and Al-Sultan (2000) considered the problem of simultaneously determining ~~the ~~optimal target means and target variances for~~ a~~ processes that might result in~~ a~~ reductions in the variability and ~~in the ~~total cost of the production process.

Costa and Rahim (2001) presented an economic design of control charts ~~with ~~considering variable sample sizes, variable sampling intervals, and variable control limits. On this basis, t~~T~~hey constructed a cost model that involves the costs of false alarms,~~,~~ ~~the cost ~~of finding and eliminating the ~~assignable ~~respective causes~~, the cost~~ associated with production for in- an out-of-control states,~~,~~ and~~ the cost~~ of sampling and testing. They assumed an exponential distribution to describe the time length ~~of time ~~the process remained~~s~~ in control for applying the Markov chain approach ~~for ~~to develop~~ing~~ the cost function.

Pignatiello and Tsai (1988) suggested a method ~~for ~~of using cost models for designing control charts when precise estimates of ~~the ~~cost model parameters are not ~~available~~possible. However, t~~T~~heir control chart designs are not sensitive to the estimates of the various cost model parameters ~~by ~~as a result of incorporating a measure of the imprecision of ~~the ~~parameter estimates into the objective function. On the other hand, Tosirisuk (1990) obtained ~~the ~~optimal process parameter control limits and process adjustment intervals that~~, which~~ minimize the total quality cost of production.

Khasawneh et al. (2008) considered the dual quality characteristics and different costs associated with each quality characteristic that falls below a lower specification limit or above an upper specification limit. Specifically, they focused on~~Khasawneh et al. (2008) considered~~ a production system that consists of a single machine and a single inspection station,~~. They~~ assuming~~ed that~~ any product has two -quality characteristics. Each product is processed and its quality characteristics ~~are ~~examined at an inspection station. A~~In their model, a~~n item is reworked~~,~~ if its performance associated with ~~the~~a quality characteristic of interest falls above an upper specification limit, scrapped if its performance falls below a lower specification limit, or accepted if its performance falls within ~~the ~~specification limits. They assumed that the rework process follows a normal distribution function similar to the initial production process. T~~t~~herefore, its process mean would be~~is~~ equal to that of the initial production process. However, in reality~~many real cases~~, the rework process typically follows different process functions that ~~should~~could ~~be considered to ~~reduce ~~the ~~total production costs. Consequently~~In this research~~, we assume that reworked items will be accepted with constant probabilities and each item may be accepted or rejected after the rework~~ed~~ process. In this case, the maximum number of rework~~ed~~ processes for any item is ~~single~~one.

Comment [A6]: Please check whether the changes I have made still convey the meaning you originally intended.

Comment [A7]: I have consolidated all discussion around Khasawneh et al. (2008) in this paragraph so that the discussion does not look disjointed.

~~1.1. Background~~

~~In a certain production process, where an item has two quality characteristics, if the value of both quality characteristics falls within the tolerance limits then the item is faultless and can be accepted. The item is considered as scrape if the value of one of its quality characteristics falls below a lower tolerance limit. Moreover, the item needs to rework, if the value of one or both quality characteristics fall above an upper tolerance limit.~~ In such a system, if the process mean is set to a low level (near ~~to~~ the lower tolerance limit), ~~then~~ the proportion of defective items~~'~~ increases and, therefore, the system experiences high scrapped item~~s'~~ costs. ~~But~~However, if the process mean is set to a high level (near the~~o~~ upper tolerance limit), ~~then~~ the proportion of items that need~~s~~ reworking increases and, thus, the system is faced with high rework cost. This situation justifies the determination of an optimum process mean (Fallahnezhad and Niaki~~,~~ 2011).

~~Khasawneh et al. (2008) considered a production system that consists of a single machine and a single inspection station. They assumed that any product has two-quality characteristics. Each product is processed and its quality characteristics are examined at an inspection station. In their model, an item is reworked, if its performance associated with the quality characteristic of interest falls above an upper specification limit, scrapped if its performance falls below a lower specification limit, or accepted if its performance falls within the specification limits. They assumed that rework process follows a normal distribution function similar to initial production process therefore, its process mean is equal to initial production process. However, in many real cases, the rework process follows different process function that should be considered to reduce the total production costs. In this research, we assume that reworked items will be accepted with constant probabilities and each item may be accepted or rejected after the reworked process. In this case, the maximum number of reworked process for any item is single.~~Specifically, i~~I~~n this ~~paper~~study, a production process with two quality characteristics is considered. A Markovian model is then developed, where defective items consisting of scraps and reworks are produced, detected, and eventually discarded during the manufacturing process~~ of manufacturing~~. To optimize~~um~~ the expected profit, scrap~~s~~ and rework~~s~~ costs are also considered in the model~~ which is discussed in~~ (see S~~s~~ection 2 for details). Subsequently, t~~T~~he optimal~~um~~ process means for the two quality characteristics are~~is~~ determined in S~~s~~ection 3. A sensitivity analysis is then performed by varying ~~the~~ cost parameters~~,~~ such as scrap~~ cost~~ and~~,~~ rework costs in S~~s~~ection 4. Finally, a~~A~~ numerical example is provided in section 5.

2. ~~The proposed~~ Model

Assume that two quality characteristics, x and y, follow a bi-variate normal distribution:~~ as is shown in equation 1.~~

Comment [A8]: The deleted phrases were duplicating previous content and were hence redundant.

$$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2+\left(\frac{y-\mu_y}{\sigma_y}\right)^2-2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right]} \quad (1)$$

Comment [A9]: Please provide editable equations and place adequate punctuation marks after each one of them. Here, for instance, the equation should be followed by a period.

In this study, ~~The notations~~ USL_x and LSL_x ~~are~~ represent the upper and lower specification limits for ~~the~~ characteristic x and ~~the notations~~ USL_y and LSL_y ~~are~~ the upper and lower specification limits for characteristic y. As previously mentioned, i~~I~~n a process, if a quality characteristic ~~was~~ is ~~less~~ below ~~than~~ its lower specification limit, then the product is considered ~~as~~ scrapped~~,~~ and if it ~~was~~ is above ~~more than~~ the upper specification limit ~~then~~ the product needs to be reworked. Other notations we use are listed below ~~defined as~~:

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c_x = ~~The c~~cost of reworking characteristic x;

c_y = ~~The c~~cost of reworking characteristic y,

c = c~~C~~ost of a scrapped item;

pc = m~~M~~anufacturing process cost for each item;

p = p~~The p~~rofit per item.

~~For~~ Consider a single-stage production system with the following states:

State 1: An item is being processed by the production system.

State 2: C~~The c~~haracteristic x is being reworked.

State 3: C~~The c~~haracteristic y is being reworked.

State 4: C~~The c~~haracteristics x and y are being reworked.

State 5: An item is accepted ~~to~~ as a ~~be~~ finished ~~work~~good.

State 6: An item is scrapped.

~~The~~ As such, the single-step transition probability matrix can be expressed as:

$$\mathbf{P}=\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ 2 & 0 & 0 & 0 & 0 & p_{25} & p_{26} \\ 3 & 0 & 0 & 0 & 0 & p_{35} & p_{36} \\ 4 & 0 & 0 & 0 & 0 & p_{45} & p_{46} \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \quad (2)$$

Comment [A10]: This equation should be followed by a comma.

where

p_{12} = p~~The p~~robability of reworking the characteristic~~s~~ x;

p_{13} = p~~The p~~robability of reworking characteristic~~s~~ y;

p_{14} = p~~The p~~robability of reworking both characteristics x and y;

p_{15} = p~~The p~~robability of accepting an item;

p_{16} = p~~The p~~robability of a scrapped item.

Moreover, for ~~the~~ characteristic x, p_{25} denotes the probabilities of accepting an item and p_{26}~~,~~ of scrapping an item after a rework process~~ed~~. For characteristic y~~,~~, the corresponding notations~~ose~~ are p_{35} and p_{36}, respectively. Finally, p_{45} and p_{46} denote the probabilities of accepting and scrapping an item after ~~its~~ both characteristics x and y are reworked.

Assuming that the quality characteristics of an item follow a bi-variate normal distribution, ~~so that~~ its probabilities can be expressed as:

$$p_{12} = \Pr\{x > USL_x, LSL_y \le y \le USL_y\} = \int_{LSL_y}^{USL_y} \int_{USL_x}^{\infty} f(x,y)\,dxdy$$

$$p_{13} = \Pr\{LSL_x \le x \le USL_x, y > USL_y\} = \int_{USL_y}^{\infty} \int_{LSL_x}^{USL_x} f(x,y)\,dxdy$$

$$p_{14} = \Pr\{x > USL_x, y > USL_y\} = \int_{USL_y}^{\infty} \int_{USL_x}^{\infty} f(x,y)\,dxdy$$

$$p_{15} = \Pr\{LSL_x \le x \le USL_x, LSL_y \le y \le USL_y\} = \int_{LSL_y}^{USL_y} \int_{LSL_x}^{USL_x} f(x,y)\,dxdy$$

$$p_{16} = \Pr\{LSL_x > x \text{ or } LSL_y > y\} = \int_{-\infty}^{LSL_y} \int_{-\infty}^{\infty} f(x,y)\,dxdy + \int_{-\infty}^{\infty} \int_{-\infty}^{LSL_x} f(x,y)\,dxdy - \int_{-\infty}^{LSL_y} \int_{-\infty}^{LSL_x} f(x,y)\,dxdy \quad (3)$$

Comment [A11]: Please place commas after all equations, except the last one, which should be followed by a period.

To analyze the absorbing Markov chain, ~~the~~ transition matrix T ~~is~~ can be rearranged as~~to~~ ~~the~~ matrix P~~ as follows~~:

$$T = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix} \quad (4)$$

Comment [A12]: This equation should be followed by a comma.

~~Therefore,~~

$$\mathbf{P}=\begin{array}{c|c} & \begin{matrix}5 & 6 & 1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}5\\6\\1\\2\\3\\4\end{matrix} & \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ p_{15} & p_{16} & 0 & p_{12} & p_{13} & p_{14}\\ p_{25} & p_{26} & 0 & 0 & 0 & 0\\ p_{35} & p_{36} & 0 & 0 & 0 & 0\\ p_{45} & p_{46} & 0 & 0 & 0 & 0\end{bmatrix}\end{array} \quad (5)$$

Comment [A13]: This equation should be followed by a period.

Fundamental matrix *M* is determined as follows:

$$\mathbf{M}=(\mathbf{I}-\mathbf{Q})^{-1}=\begin{bmatrix}1 & -p_{12} & -p_{13} & -p_{14}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}^{-1}=\begin{bmatrix}1 & p_{12} & p_{13} & p_{14}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (6)$$

Comment [A14]: This equation should be followed by a period.

A~~The a~~bsorption probability matrix F is determined as follows (Bowling et al. 2004):

$$\mathbf{F}=\mathbf{M}\times\mathbf{R}=\begin{bmatrix}1 & p_{12} & p_{13} & p_{14}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}p_{15} & p_{16}\\ p_{25} & p_{26}\\ p_{35} & p_{36}\\ p_{45} & p_{46}\end{bmatrix}=$$
$$\begin{bmatrix}p_{15}+p_{12}p_{25}+p_{13}p_{35}+p_{14}p_{45} & p_{16}+p_{12}p_{26}+p_{13}p_{36}+p_{14}p_{46}\\ p_{25} & p_{26}\\ p_{35} & p_{36}\\ p_{45} & p_{46}\end{bmatrix} \quad (7)$$

Comment [A15]: This equation should be followed by a comma.

w~~W~~here f_{15} and f_{16} are the probabilities of accepting ~~and~~ or scrapping ~~one~~ an item, respectively.

Now, the expected profit per item can be defined as:

$$E(PR)=f_{15}p-pc-p_{12}c_x-p_{13}c_y-p_{14}(c_x+c_y)-f_{16}c \quad (8)$$

Comment [A16]: This equation should be followed by a period.

~~With~~ By substituting~~,~~ f_{15} by the optimal values of μ_x~~,~~ and μ_y~~,~~ ~~that~~ maximizes ~~the~~ expected profit: ~~can be reached.~~

$$E(PR)=(p_{15}+p_{12}p_{25}+p_{13}p_{35}+p_{14}p_{45})p-pc-p_{12}c_x-p_{13}c_y-$$
$$p_{14}(c_x+c_y)-(p_{16}+p_{12}p_{26}+p_{13}p_{36}+p_{14}p_{46})c \quad (9)$$

Comment [A17]: This equation should be followed by a period.

3. Numerical Example

Consider ~~ing~~ a single-stage production system with the following parameters:

$p = 120,\ pc = 45,\ c_x = 15,\ c_y = 12,\ c = 14, \sigma_x = 1, \sigma_y = 1, \rho = 0,\ L_x = 8.0,\ L_y = 13.0,$
$U_x = 12.0$ and $U_y = 17.0$.

Furthermore, from ~~the~~ historical data, we have $p_{25} = 0.9$, $p_{35} = 0.9$, and $p_{45} = 0.8$. Using the search procedure by plotting *E(PR)* as a function of μ_x and ~~,~~ μ_y, we can~~it is~~ conclude~~d~~ that ~~the~~ expected profit is maximized at $\mu_x = 10.4$ and~~,~~ $\mu_y = 15.4$ ~~with~~ by ~~an~~ amount ~~of~~ 69.88. The optimal values of μ_x and~~,~~ μ_y are determined by evaluating the value of the objective function for different values of μ_x and~~,~~ μ_y.

The expected profit as a function of ~~the~~ process means (μ_x, μ_y) is shown in F~~f~~igure 1.

Insert Figure 1

The expected profit is a concave function over ~~the~~ interval [($L_x = 8$, $U_x = 12$), ($L_y = 13$, $U_y = 17$)]. S~~A s~~ensitivity analysis is then performed to analyze the possible impact of ~~the~~ rework and scrap costs on ~~the~~ optimal process means and ~~the~~ optimal expected profit. The effects of ~~different~~ the various rework~~ed~~ and scrap costs ~~at~~ for each state are shown in ~~the~~ Table 1.

Insert Table 1

From Table 1 ~~indicates that~~, by increasing the value of cx~~-~~, the optimum value of μ_x decreases. However,~~,~~ ~~but it does not affect on~~ the optimum value of μ_y ~~also~~ is not affected by increases in~~ing~~ the value of c_y~~-~~. In other words, as~~,~~ the optimum value of μ_y decreases, ~~but simultaneously~~ the optimum value of μ_x slightly and simultaneously increases.~~,~~ ~~A~~Overall,~~lso~~ changing~~-~~ the value of *c* changes the optimum values of μ_x and~~,~~ μ_y.

4. Conclusions

In this ~~paper~~study, a production process with two quality characteristics has been considered and a~~. A~~ Markovian model ~~is~~ developed for determining the ~~the~~ expected profit by considering processing, scrap, and rework costs. One numerical example was then provided to illustrate the applications of the proposed models. The results showed that the rework and scrap costs for both quality characteristics ~~extremely~~ a~~e~~ffect~~s~~ ~~on the~~ optimal process means significantly.

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Comment [A18]: Please check whether you agree with these notation changes.

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